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Article

Study on the Influence of Multiple Derivation Method of General Term Formula of Series on Students' Logical Ability

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Abstract: This paper discusses the influence of multiple derivation method on students' logical reasoning ability. By comparing traditional derivation method with multivariate derivation method, this paper analyzes how the latter can improve students' flexibility and accuracy in solving problems through various strategies such as graphic method, induction method and recurrence method. The research shows that adopting multiple derivation method can strengthen students' logical reasoning ability, enhance the depth and creativity of students' mathematical thinking, and promote students' multi-angle thinking divergence when solving mathematical problems. This method helps students better understand the knowledge of number series, and exercises their rigorous logical thinking and the ability to solve practical problems. It opens up a new teaching idea and operation direction for mathematics teaching.

Keywords: series general term formula; multivariate derivation method; logical ability; mathematics teaching; reasoning ability

1. Introduction

In the field of mathematics teaching, it is a key link to train students' mathematical logical thinking to explore the derivation of general term formula of series. Traditional derivation methods often adopt a single model, which has limitations in stimulating students' logical reasoning and innovative consciousness. Recently, more and more attention has been paid to the multivariate derivation method, which combines graphic method, induction method, recurrence method and other skills, and opens up a variety of ideas for students to solve problems. This method increases students' flexibility in solving problems and helps to strengthen students' reasoning ability. The purpose of this paper is to provide theoretical support for mathematics teaching activities and to provide reference for teaching innovation in education reform.

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2. Overview of Multivariate Derivation Methods for General Term Formulas of Series

2.1. Definition and Classification of General Term Formulas of Series

The general term formula expresses the relationship between the NTH term and its position in the series, so that students can directly calculate any term according to the specified number of terms. This formula is a core tool in the study of series and is essential for developing students' abstract thinking and reasoning skills. According to the different properties of the series, the general term formula can be divided into the following categories:

Arithmetic series: Arithmetic series is characterized by a constant difference between two adjacent terms, known as tolerance d. The general term formula is:

$$a_n = a_1 + (n-1) \cdot d \quad (1)$$

Where, a_1 is the first item, d is the tolerance, and n is the number of items. Arithmetic series has a strong linear character, which is often seen in the arithmetic difference problem in daily life.

Geometric sequence: The characteristic of geometric sequence is that the ratio of adjacent terms is constant, called the common ratio r, and its general formula is:

$$a_n = a_1 \cdot r^{(n-1)} \quad (2)$$

Where a_1 is the first term, r is the common ratio, and n is the number of terms. Geometric series is widely used in interest, population growth and other problems, showing the characteristics of exponential growth or decline.

Recursive sequence: Each term of a recursive sequence has a recursive relationship with the previous terms, usually expressed by a recursive formula, such as the Fibonacci sequence:

$$F_n = F_{n-1} + F_{n-2}, F_1 = 1, F_2 = 1 \quad (3)$$

General-term formulations of recursive sequences often require a combination of mathematical induction or other advanced techniques.

Special series: including harmonic series, square series, cubic series, etc. Common general term formulas are:

$$a_n = \frac{1}{n}, a_n = n^2, a_n = n^3 \quad (4)$$

These sequences often appear in specialized mathematical problems or practical application scenarios. There are many kinds of classification and derivation methods for general term formulas of series of numbers. It is of great benefit to students to use various mathematical methods to solve practical problems to master the characteristics and internal relations of various series of numbers.

2.2. Comparison between Traditional Derivation Methods and Multivariate Derivation Methods

The traditional derivation methods of general term formula of series often rely on a single inference path, such as algebraic method, difference method and recursion method. These methods focus on the direct application of formulas or the repetition of a pattern. They are suitable for solving basic series problems, but they are not flexible enough. This practice may distract students from other problem-solving approaches and affect their overall logical thinking ability. In contrast, the multiple derivation method combined with graphic method, induction method and recursion method can dig deeply the law of series from multiple angles, and guide students to understand the problem with various ways of thinking. This method can strengthen students' logical reasoning ability and promote them to show more diversity and creativity in the process of solving problems. Figure 1 below shows how traditional derivation methods compare to multivariate derivation methods:

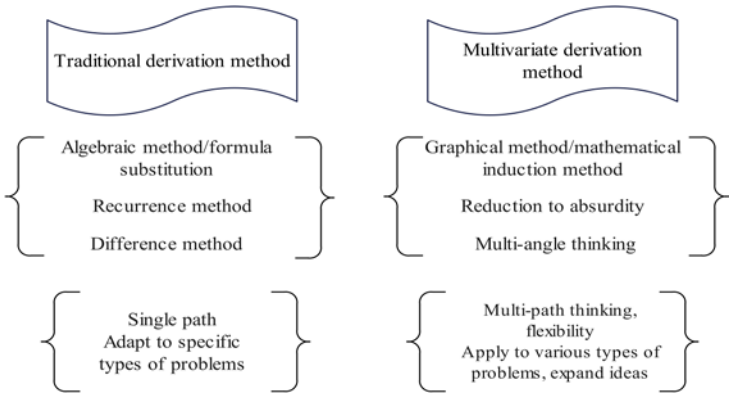

Fig. 1 Comparison between traditional derivation methods and multivariate derivation methods

2.3. Connotation of Multivariate Derivation Method

The multiple derivation method involves the comprehensive application of different strategies from multiple perspectives, so that students can have a comprehensive grasp of the pattern of the series and can derive the general term formula efficiently. The key of this method is to combine a variety of mathematical tools and ways of thinking, so that students can deeply analyze series problems from different perspectives, and enhance students' mathematical reasoning ability. In the multivariate derivation method, the graphical method plays an indispensable role, which can clearly show the organization structure and development trend of the series with the help of images or tables. For example, it is easier for students to identify the law of the series with the help of graphics to depict the number line according to the decreasing characteristics of the series. This visual processing method helps enhance students' intuitive understanding and mastery of the law of the series. In addition, mathematical induction, as a core skill in multivariate derivation, is in a critical position.

Through induction, students can derive and verify the accuracy of formulas step by step, thus improving the logic of thinking and reasoning ability. Recursion method and disproof rule provide effective tools for solving complex series problems. The recursion method gradually deduces other terms through the initial term and the recursion relation of the series, while the counter-proof rule is used to verify the accuracy of the formula or reveal the contradiction of the hypothesis, so as to exercise students' logical reasoning ability. The multiple derivation method not only improves the skills of understanding problems, but also cultivates the students' ability to flexibly and creatively solve problems, so that they can choose the most appropriate derivation strategy when facing complex mathematical problems.

3. Influence of Multiple Derivation Method of General Term Formula of Series on Students' Logical Ability

3.1. Improve the Diversity and Accuracy of Problem-Solving Strategies

Using multiple derivation methods, students can solve series problems from different angles through a variety of mathematical tools and ways of thinking, and improve the diversity and accuracy of problem understanding strategies. Compared with the traditional single method, such as algebraic method or recursive method, the multivariate derivation method encourages students to analyze from multiple angles and helps them flexibly choose a more suitable solution path. For example, students can use algebraic methods to derive general term formulas, use graphical methods to show sequence laws, or test hypotheses by induction. This variety of problem-solving ideas not only enhances students' self-confidence, but also greatly improves the efficiency and accuracy of understanding the questions.Table 1 below summarizes the characteristics and application scenarios of different derivation methods:

Table 1. Features and application scenarios of different derivation methods.

Derivation method	peculiarity	Application scenario
Algebraic method	Through algebraic calculation, the structure is clear and the steps are simple	Arithmetical series, geometric series, etc
Graphic method	Show the growth trend or pattern of the series by graph or table	It is suitable for scenes that visually display sequence features
induction	It is derived step by step through hypothesis and verification, and the logic is rigorous	Recursive sequence, sequence of complex laws

Recurrence method	Using recursion formula to deduce one term by one term, it is suitable to deal with the sequence of numbers with obvious recursion relationship	A recurrence sequence or sequence in which the recurrence relationship is clear

3.2. Enhance Students' Logical Reasoning Ability and the Rigor of Reasoning Thinking

By adopting a variety of derivation methods for the general term formula of the series, this process integrates various ways of thinking, and significantly improves students' logical reasoning ability and the rigor of reasoning thinking. Compared with the previous single derivation path, this diversified derivation method pays more attention to guiding students to conduct in-depth exploration from multiple perspectives, and requires students to use more complex mathematical tools and ways of thinking.

This process puts the students' logical reasoning skills to the test and strengthens their logical rigor of the reasoning steps. In mathematical induction, for example, students must verify the initial case and derive a general formula based on inductive assumptions, thus guaranteeing the accuracy of the reasoning at each step. For the recurrence method, students need to conduct an in-depth analysis of the recurrence relationship and gradually deduce the subsequent terms, which is helpful to improve the comprehensiveness of the derivation process and enhance its logic. Table 2 below summarizes the influence of different derivation methods on students' logical reasoning ability and the rigor of reasoning thinking:

Table 2. Influences of derivation methods on logical reasoning ability and the rigor of derivation thinking.

Derivation method	Effect on logical reasoning ability	Influence on the rigor of reasoning thinking
Mathematical induction	To enhance students' reasoning ability from the local to the whole requires rigorous verification of hypotheses	Improve the hierarchy of thinking and ensure that each step of the derivation has rigor
Recurrence method	Improve students' understanding and reasoning ability of recurrence relation	Enhance step-by-step verification and logical coherence in the derivation process
Graphic method	Help students intuitively understand the law of series, enhance the intuitiveness of logical reasoning	Enhance the derivation and verification of sequence law, and verify the derivation process through graphics

3.3. Promote the Flexibility and Innovation of Students' Mathematical Thinking

Multiple derivation method integrates all kinds of mathematical tools and thinking modes, guides students to analyze series problems in various directions, broadens the way of thinking and improves the accuracy. Traditional derivation methods are limited to a single idea, such as algebraic method or recursive method, and students often feel confused when dealing with complex series. The multivariate derivation method focuses on multi-angle analysis, enhances the flexibility of understanding questions, and helps students find the appropriate deduction path. Students can deduce the general term formula by means of analytic geometry, and at the same time demonstrate the regularity of number series visually by means of graphics. In addition, they can also test the correctness of propositions by means of inductive reasoning. Each of these tools has its own characteristics, so that students can deal with problems flexibly, improve the rate of understanding and confidence. Table 3 below summarizes the role of multiple derivation methods in promoting the flexibility and innovation of students' mathematical thinking:

Table 3. The role of multiple derivation methods in promoting the flexibility and innovation of mathematical thinking.

Derivation method	To promote the flexibility of mathematical thinking	Promotion of innovation
Graphic method	Provides a way to visually display the rules of the series and enhance flexibility	Stimulate students' innovative thinking through the variety of graphics
Algebraic method	Emphasis on algebraic analysis of series, improve the flexibility of solving problems	Stimulate students to innovate solutions through different algebraic operations
induction	Enhanced flexibility from hypothesis to derivation	Students are encouraged to come up with new hypotheses and test their reasoning
Recurrence method	Through the discovery of recurrence relations, the flexibility of solving problems is increased	Stimulate students to explore the deeper laws behind recursive relationships

4. Application of Multiple Derivation Method of General Term Formula in Mathematics Teaching

4.1. The Combination of Graphic Method and Formula Deduction Method Cultivates Logical Thinking

The combination of graph method and formula derivation method has a remarkable effect on the improvement of students' logical thinking ability when deriving the general term formula of series. The graphic method presents the pattern of the series with the help of vivid graphics, so that students can intuitively grasp the relationship between the series. Formula deduction requires algebraic logic to provide an accurate mathematical representation of this pattern. The combination of these two methods can help students understand the pattern of number series deeply and enhance their rigorous logical reasoning ability. Taking the arithmetic number as an example, suppose that the first item of the series is a_1, the tolerance is d, and the general formula is:

$$a_n = a_1 + (n-1)\cdot d \quad (5)$$

Using the graphical method, students can visually represent the terms of a series on a number line and observe that the difference between each term and the previous term is always a constant d. For example, if $a_1 = 3$ and tolerance d=2, the first items in the sequence are: 3,5,7,9,11,..... Students can intuitively see that the space between each item is 2 by marking these values on the number line.

After mastering the increase and decrease pattern of the series, students can deduce the general term formula of the series by means of formula deduction. This derivation process allows students to understand how each term in the sequence is generated according to the first term and tolerance d, and exercises students' ability to find regularities by algebraic methods.

4.2. Induction and Contradiction Promote Logical Reasoning Ability

In the field of mathematics, inductive reasoning and reverse proof are two key ways of reasoning and argumentation, which help to strengthen students' logical reasoning ability when guiding students to deduce the general term formula of a series of numbers. In particular, inductive reasoning shows its unique advantage in verifying the correctness of the general term formula of series, especially when the recurrence relation of series is more complicated, inductive reasoning is particularly useful. By induction, it is proved

that the formula is valid for all natural numbers n=k by verifying the initial case and assuming that the formula is valid for a certain natural number n=k, and then it is deduced that the formula is also valid for y and n=k+1. Taking the equal ratio as an example, assume that the common ratio of a_1, a_2, a_3,.....of the series is r, and its general term formula is:

$$a_n = a_1 \cdot r^{n-1} \quad (6)$$

First verify the basic case: n=1, the formula is established, that is:

$$a_1 = a_1 \cdot r^{1-1} = a_1 \quad (7)$$

Induction hypothesis: Suppose that for n=k, the formula holds, i.e. :

$$a_k = a_1 \cdot r^{k-1} \quad (8)$$

Induction step: Prove that for n=k+1, the formula holds, that is:

$$a_{k+1} = a_1 \cdot r^{(k+1)-1} = a_1 \cdot r^k \quad (9)$$

Through the induction step, the conclusion is drawn that for all $n \in N$, the formula $a_n = a_1 \cdot r^{n-1}$ holds, thus proving the general term formula of the series.

Paradoxical proof indirectly proves the truth of a proposition by assuming that the conclusion is not true and deducing the contradiction. In series verification, students can assume that the series does not conform to the general term formula and derive results that contradict the known conditions, thus confirming the accuracy of the formula. This method helps students exercise their logical reasoning skills from the negation of hypotheses, and enhance their understanding of the rigor and depth of reasoning.

4.3. The Combination of Recursion Method and Sequence Law Improves the Depth of Thinking

Recursion method is a common technique in series derivation, which helps students to derive unknown terms step by step from known terms, so as to deepen the understanding of series rules. It emphasizes the logical progression between known and unknown, encourages students to discover the fundamental laws of series in the process of derivation, and improves the ability of mathematical logical thinking. The recursion method is especially suitable for the sequence of numbers without definite general term formula, and the law of sequence can be revealed by recursion formula. For example, the recurrence formula for the Fibonacci sequence is:

$$F_n = F_{n-1} + F_{n-2}, F_1 = 1, F_2 = 2 \quad (10)$$

By recursion method, students can start from the initial term and gradually calculate the subsequent terms of the series. For example, when calculating F_3:

$$F_3 = F_2 + F_1 = 1 + 1 = 2 \quad (11)$$

Then, calculate F_4:

$$F_4 = F_3 + F_2 = 2 + 1 = 3 \quad (12)$$

Further recursion leads to terms such as $F_5 = 5$, $F_6 = 8$. Through this recursive process, students not only master the calculation method of the series, but also perceive the law of the growth of the series.

Another important application of recursion is to help students understand complex recursion relationships. For example, consider a weighted average series whose recurrence formula is:

$$a_n = \frac{1}{2}(a_{n-1} + a_{n-2}), a_1 = 1, a_2 = 2 \quad (13)$$

By recursion, students can calculate subsequent sequence items, such as:

$$a_3 = \frac{1}{2}(a_2 + a_1) = \frac{1}{2}(2 + 1) = 1.5 \quad (14)$$

$$a_4 = \frac{1}{2}(a_3 + a_2) = \frac{1}{2}(1.5 + 2) = 1.75 \quad (15)$$

Through the recursive method, students can gradually calculate the subsequent items of the series, and exercise the ability of mathematical thinking and logical reasoning.

Combined with the recurrence law of series, students can not only skillfully apply the recurrence method to solve practical problems, but also deepen the understanding of series law. This kind of training broadens the depth and breadth of students' thinking, and improves their resilience in the face of complex problems.

5. Conclusion

The variety of formula derivation methods of series of general terms has injected new vitality into the mathematics course and effectively promoted the improvement of students' logical thinking level. Through the integration of graphical method, induction method, contrarian method and recursion method, students can understand the essential law of number series from multiple dimensions, and then cultivate rigorous logical reasoning ability and creative thinking. These methods enable students to deal with various problems more flexibly, optimize the process of solving problems, and deepen the understanding of mathematical laws. The multiple derivation method of the general term formula of series is not only a tool of mathematical knowledge, but also a key way to promote students' thinking progress and improve their mathematical literacy.

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